1998-99 CATS ASSESSMENT
Open-Response Item Scoring Worksheet

Grade 8—Mathematics

The **academic expectations** addressed by "Alex's Garden" are

2.7 Students understand number concepts and use numbers appropriately and accurately.

2.10 Students understand measurement concepts and use measurements appropriately and accurately.

The **core content** assessed by this item includes

Geometry/Measurement Skills

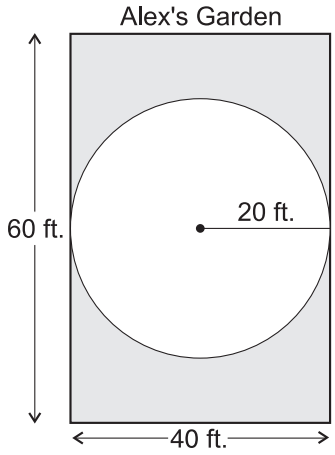
- Students should be able to use formulas to find measurements of two-dimensional shapes.

Number/Computation Skills

- Students should be able to add subtract, multiply and divide rational numbers (fractions, decimals, percents) and integers.

Alex's Garden

Alex is watering part of his garden with a sprinkler that covers a circular area shown in the diagram below.



a. What is the total **area** of Alex's garden? Show your work.

b. What is the **area** of the part of the garden that is being watered by the sprinkler? Express your answer to the nearest square foot. Show your work.

c. What **percent** of his garden is being watered by the sprinkler? Express your answer to the nearest percent. Show your work.

BE SURE TO LABEL YOUR RESPONSES (a), (b), and (c).

SCORING GUIDE
Grade 8 Mathematics

Score	Description
4	Student answers parts a, b, and c correctly and shows work.
3	Student answers two of three parts correctly with the third part incorrect due to a calculation error or rounding error with work shown. **OR** Student correctly answers three parts with some work shown.
2	Student correctly answers two or three parts with some or no work shown. **OR** Student answers part c correctly based on parts a and b which contain calculation or rounding errors only.
1	Student answers part a or part b correctly with or without work shown. **OR** Student shows some understanding of the problem (i.e., part c correct based on calculation or rounding errors in a and b).
0	Response is totally incorrect or irrelevant.
Blank	No response.

Correct answers:

Part a. 40 x 60 = 2400 sq. ft.

Part b. Area = 1256 or 1257 sq. ft., either answer is acceptable depending on the value of pi.

Part c. Show or explain that the nearest percent for 1256/2400 or 1257/2400 is 52%.

Note: Correct labels needed at "4" level only.

ANNOTATED STUDENT RESPONSE
Grade 8 Mathematics

Sample 4-Point Response of Student Work

Student Response

A. Total Area

$$\begin{array}{r} 60 \\ \times 40 \\ \hline 2400 \end{array}$$

2400 sq.ft.

B. Watered Area

$\pi \times 20^2$
3.1415×400
$1257\ ft^2$

C. $\frac{1257}{2400} = \frac{x}{100}$

$\frac{125700}{2400} = \frac{2400x}{2400}$

$x = 52.375$

$\approx 52\%$

The total area of her garden is 2400 sq. ft. The area of her garden that is being watered is 1257 sq. ft. The percent of her garden that is being watered by the sprinkler is approximately 52%.

← Student finds the correct answer for part a and shows the process used to calculate the area (i.e., 60 x 40 = 2400).

← Student shows the process used to calculate the area of the part of the garden that is being watered by the sprinkler (i.e., $\pi \cdot 20^2$).

← Student correctly answers that the area of the part of the garden that is being watered by the sprinkler, rounded to the nearest foot, is 1257 ft^2.

← Student shows the process used to calculate the percent of the garden being watered (i.e., $\frac{1257}{2400} = \frac{x}{100}$).

← Student correctly answers that the percent of the garden being watered, rounded to the nearest percent, is 52%.

Overall, the student demonstrates a solid understanding of measurement concepts by correctly using formulas to find measurements of two-dimensional shapes. In addition, the student demonstrates a strong understanding of percents.

ANNOTATED STUDENT RESPONSE
Grade 8 Mathematics

Sample 4-Point Response of Student Work

Student Response

A. The total area of Alex's garden is 2400 sq ft. (2,400 ft.^{2})

$L \cdot W = 60 \cdot 40 = 2400$

B. The total area of the garden where the sprinkler can reach is 1256 ft^2.

$3.14 \cdot 20^2$

$3.14 \cdot 400 = 1{,}256$

C. $1{,}256 \div 2400 = 0.52\overline{3}$

$\cong 52\%$ of the garden is being watered by the sprinkler.

← Student finds the correct answer for part a and shows the process used to calculate the area (i.e., $60 \cdot 40 = 2400$).

← Student correctly answers that the area of the part of the garden where the sprinkler can reach, rounded to the nearest foot, is 1256 ft.^{2}

← Student shows the process used to calculate the area of the part of the garden that is being watered by the sprinkler (i.e., $\pi \cdot 20^2$).

← Student shows the process used to calculate the percent of the garden being watered by the sprinkler (i.e., $1{,}256 \div 4205 = 0.52\overline{3}$).

← Student correctly answers that the percent of the garden being watered, rounded to the nearest percent, is 52%.

Overall, the student demonstrates a solid understanding of measurement concepts by correctly using formulas to find measurements of two-dimensional shapes. In addition, the student demonstrates a strong understanding of percents.

ANNOTATED STUDENT RESPONSE
Grade 8 Mathematics

Sample 3-Point Response of Student Work

Student Response

A. Total area of Alex's garden.

$A = lw$

$A = (60)(40)$

$A = 2400 \text{ ft.}^2$

B. Area of the circular part.

$A = \pi r^2$

$A = 3.14(20)^2$

$A = 3.14(400)$

$A = 1256 \text{ ft.}^2$

C. Percent being watered by the sprinkler

$1256 \div 2400 = .52\overline{3} \times 100 = 52.\overline{3} = 52.\overline{3}\%$

← Student finds the correct answer for part a and shows the process used to calculate the area (i.e., (60) (40) = 2400).

← Student shows the process used to calculate the area of the part of the garden that is being watered by the sprinkler (i.e., $3.14 \cdot 20^2$).

← Student correctly answers that the area of the part of the garden that is being watered by the sprinkler, rounded to the nearest foot, is 1256 ft².

← Student shows the process used to calculate the percent of the garden being watered by the sprinkler (i.e., $\frac{1256}{2400} = \frac{x}{100}$).

However, answer is incorrect due to rounding to the nearest tenth of a percent rather than to the nearest percent.

Overall, the student demonstrates a strong understanding of measurement concepts by correctly using formulas to find measurements of two-dimensional shapes. In addition, the student shows some understanding of percents, but the work contains rounding errors.

ANNOTATED STUDENT RESPONSE
Grade 8 Mathematics

Sample 2-Point Response of Student Work

Student finds the correct answer for part a and shows the process used to calculate the area. However, the student does not label the answer.

Student correctly answers that the area of the part of the garden that is being watered by the sprinkler, rounded to the nearest foot, is 1256. However the student does not label the answer.

Student incorrectly answers that the percent of the garden being watered by the sprinkler is 50%. No work is shown.

Student Response

A. The total area of Alex's garden is 2400. Because area = length x width

60 x 40 = 2400

B. Circle area = p · radius^2

3.14 x 20^2 = 1256

C. 50%

Student shows the process used to calculate the area of the part of the garden that is being watered by the sprinkler (i.e., π•radius^2). Student uses 3.14 as the value for π and 20 as the radius.

Overall, the student shows some understanding of measurement concepts and a limited understanding of percents.

ANNOTATED STUDENT RESPONSE
Grade 8 Mathematics

Sample 1-Point Response of Student Work

Student Response

A. 60
x 40
2400

The total area of Alex's garden is 2400 feet.

B. 3.14 x (20 x 2)
3.14 x 40 = 125.6
126

With my answer expressed to the nearest square foot 126 feet of Alex's garden is being watered by a sprinkler.

C. 126 ÷ 100 = 12.6
13%

With my answer expressed to the nearest percent, 13% of Alex's garden is being watered by a sprinkler.

← Student finds the correct answer for part a and shows the process used to calculate the area (i.e., 60 x 40 = 2400). However, the student incorrectly labels the answer (i.e., feet).

← Student shows incorrect process to determine the area of the garden that is being watered by the sprinkler (i.e., instead of 20^2, uses 20 x 2).

← Student uses an incorrect procedure to determine the percent of the garden. Student shows some understanding of percent by dividing by 100, however student simply divides the area in part b by 100.

Overall, the student shows minimal understanding of measurement concepts and percents.

INSTRUCTIONAL STRATEGIES
Grade 8 Mathematics

The open-response item **"Alex's Garden"** assesses students' ability to use formulas to find measurements (i.e., area) of two-dimensional shapes. In addition, it assesses students' ability to determine percent based on a geometric model. The instructional strategies below present ideas for helping students explore and master these concepts.

Using concrete and representational models (e.g., geoboards, dot/graph paper), review the concept of area.

Review the formulas used to determine the area of a variety of two-dimensional shapes.

Using concrete and representational models demonstrate the relationship of part to whole to determine percent.

Review formulas and methods used to determine percent.

Provide students with multiple opportunities to work on a variety of math problems that require them to decipher and organize information (e.g., CATS-like open-response items). Teach students a variety of strategies for organizing information (e.g., using tables, charts, graphs, highlighting and underlining strategies) as they solve problems and make comparisons or show relationships. Such opportunities can help students learn to use information to write equations, check their reasoning and the reasonableness of their answers, document their thinking, and explain their work to others.

Provide opportunities for students to work individually, in pairs, in small groups, and/or as a class to complete (with teacher guidance and support) any or all of the following activities:

- Using concrete models (e.g., geoboards or pattern blocks), practice calculating areas of a variety of shapes.
- Using formulas and a calculator, practice calculating areas of a variety of shapes.
- Practice finding area of exposed or unexposed space when one shape is placed over another.
- Practice calculating percent of exposed or unexposed space when one shape is placed over another.
- Practice rounding.
- Discuss and write about strategies for calculating areas of exposed or unexposed areas when one shape is placed over another. This can help students develop and/or refine their ability to effectively communicate their mathematical thinking both verbally and in writing. Prior to these activities, the teacher can model strategies for communicating mathematical thinking.
- Review correct labelling for area of two-dimensional shapes.